Think About This Situation

A popular game held at fairs or parties is the jelly bean guessing contest. Someone fills a jar or other large transparent container with a known quantity of jelly beans and gives a prize to the person who guesses closest to the actual number. Participant estimates often vary and rarely does the winner guess the actual number of jelly beans.

Suppose you and your friend are going to enter the "*Jelly Bean Counting Contest*" at the county fair.

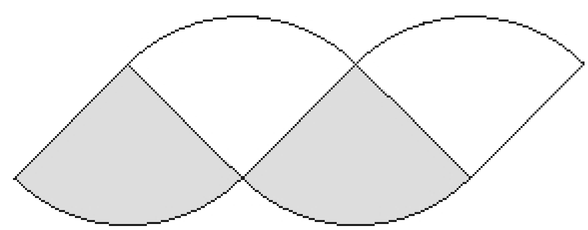
a) Consider the bucket of jelly beans. How many jelly beans do you think are in the bucket?
b) What strategies did you use to arrive at your estimate?
c) How might your strategy change if you could touch the bucket but not take the jelly beans out?

Investigation: Discovering Formulas for Area & Volume

The world in which we live is three-dimensional. It is filled with things that take up space and there is a need to describe how much space is being used. Length, area, and volume are ways to describe the size of different shapes. For simple shapes, such as cylinders and pyramids, formulas can be used to calculate the size measures of area and volume. For other shapes, like buildings or vases, finding those size measures require a little more investigation into what information and calculations are needed. As you work on the problems in this investigation, keep track of strategies used to answer the question:

How are the formulas for area and volume developed?

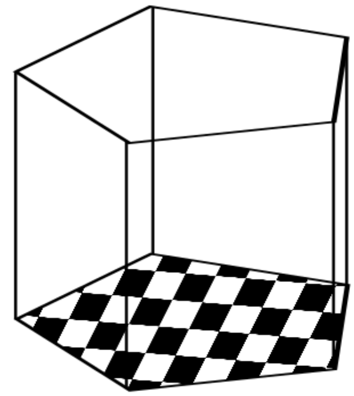
1. The formula for the area of a circle is $A = \pi r^2$ but where does that formula come from?
   a. Consider splitting any size circle into pieces and rearranging them into a recognizable shape. Start by cutting one of the circles from the handout into fourths and arranging the pieces as illustrated in the figure to the right. Each group member should use a different size circle. Is this new figure a familiar shape?



   b. Have each person in your group divide their circle pieces into smaller portions such $\frac{1}{8}\,or\,\frac{1}{16}$ and rearrange the pieces. What is the approximate shape of the new figure? Describe how to calculate the area of the new figure.
   c. What measurements are needed to complete the calculations described in part b? How do those measurements relate to the original circle?
   d. Explain how the area of the new figure relates to the area of the original circle.
   e. If the area of a circle is $A = \pi r^2$, how does that compare to the calculations you described in part b? Justify your response.

2. The measurement of area is commonly given in square units. Volume is commonly reported as cubic units. Discuss and illustrate how cubic units and square units are similar and how they are different.

3. Volume is a measurement of the physical space used by an object.
   a. Look at the prism to the right and estimate the area of the base.
   b. How many unit cubes would completely cover the base? Explain your reasoning.
   c. If there are 3 layers of unit cubes, what is the volume? What is the volume if there are 130 layers?
   d. Suppose a different figure has a base area of 13 square units. What is the volume if there are *h* layers of unit cubes?
   e. Describe the information needed and calculations used to find the volume of a prism.
   f. Write a formula for calculating the volume *V* of a prism which has a base area *B* square units and a height of *h* unit layers.
   g. A common formula for the volume of a rectangular prism is $V = (length)(width)(height)$. How does that compare to the formula you wrote in part f?



4. Some products like vegetables or sodas are packaged in cylindrical cans.
   a. How are cylinders and prisms similar and how are they different?
   b. Give an argument for why the volume of a cylinder is $V = \pi r^2 h$.

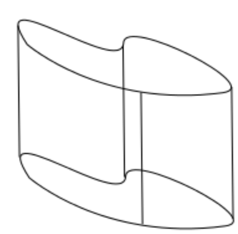
5. Revisit the Jelly Bean Counting Contest and consider that the dimensions of the bucket are 7 cm in diameter and 10 cm in height.
   a. How many jelly beans would cover the bottom of the bucket in a single layer if each jelly bean were 1.5 cm long and 1 cm wide?
   b. How many layers of jelly beans could fit in the bucket? Explain your reasoning.
   c. What is the volume of the bucket in jelly beans? Is this a reasonable estimate for the number of jelly beans in the bucket? Explain.

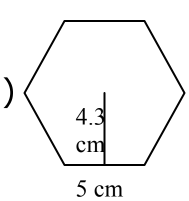
Summarize the Mathematics

a) Describe how the dissection of a circle helped in developing a formula for the area of a circle.
b) How is the formula for the volume of a cylinder similar to the formula for the volume of a prism?
c) Give a reason that would convince someone that the formula $V = \pi r^2 h$ is the formula for the volume of a cylinder.

Check Your Understanding

a) If the figure to the right has a base area of 14 in² and a height of 12 in, what is the volume?
b) Use a dissection technique to find the area of the regular hexagon. (A regular hexagon has all sides the same length.)





Investigation: Discovering Formulas for Area & Volume (Teacher Notes)

NC CCSS Math 1: Quantities

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and origin in graphs and data displays.

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.

NC CCSS Math 1: Seeing Structure in Expressions

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*

NC CCSS Math 1: Geometric Measurement & Dimension

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

G-GMD.2 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Think About this Situation

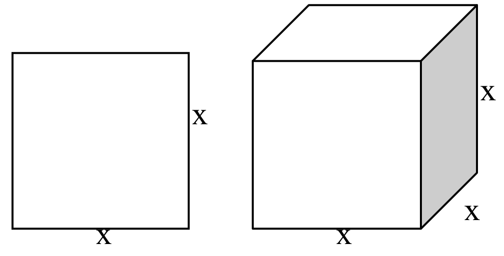
Consider using an actual container of jelly beans. If not, use the picture provided to generate discussion about the situation.

a) Student answers may vary.
b) Students may use a variety of strategies. Have students discuss and compare their strategies. Have students critique the reasoning of others by commenting on a strategy others used. Would they change the strategy they used for another's strategy? Why or why not?
c) Students may consider actually measuring the bucket. What measurements would be helpful? How would they use the measurements?

Investigation: Discovering Formulas for Area and Volume

1.
   a. In order to get fourths, students will need to locate the center of the circle. This was purposely left off of the diagram to allow students to think about ways to get the center. One method is to fold the circle in half, open it and fold in half again, thus creating the center where two diagonals meet. After cutting and rearranging students may respond that there is not a recognizable shape. The sections may be too big to see the similarity to a parallelogram.
   b. As the pieces become smaller portions, the shape of the new figure should resemble a parallelogram. Students should use the area formula for a parallelogram (A=bh) for the estimated area. The radius of the circle should be used as the height of the parallelogram. Half of the circumference should be used to estimate the base of the parallelogram.
   c. The base and height of the parallelogram are estimated by ½ the circumference and the radius of the circle.
   d. The area of the new figure is the same as the area of the circle.
   e. The area of the parallelogram is A=bh. If $b=\frac{1}{2}(2\pi r)=\pi r$ and $h = r$ then $A=bh=(\pi r)r=\pi r^2$

2. A square unit has two linear dimensions which is illustrated by a square with side length x. The area of the square is x^2. A cubic unit has three linear dimensions which is illustrated by a cube with side lengths x. The volume of a cube is x^3.



3.

a. The base has an estimated area of 37 square units.
b. 37 unit cubes should cover the base since each unit cube fits on one unit square.
c. 3 layers of 37 unit cubes per layer would be 111 unit cubes; 130 layers of 37 unit cubes per layer would be 4810 unit cubes
d. V=13h
e. The area of the base and the height of the prism can multiplied to calculate the volume of a prism.
f. V = Bh
g. The area of the base of a rectangular prism is *(length)(width)*. Since B=*(length)(width)* then the volume of a rectangular prism can also be expressed as V=Bh.

4.

a. Both cylinders and prisms have two bases. However, the cylinder base is a circle and the prism base is a polygon. The sides of a prism are multiple rectangles, whereas, the side of a cylinder can be modeled by a single rectangle going around the circumference of the circle.
b. The area of the circle is πr^2. Substitute the area of a circle for the base area of a cylinder B and $V = \pi r^2 h$.

5.

a. The base of the bucket has an area of $\pi(3.5\text{ cm})^2 = 38.5\text{ cm}^2$ The jelly bean (if positioned laying on the longest side and not standing on end) would cover approximately (1.5 cm)(1 cm)= 1.5cm^2. The area of the base divided into 'jelly bean' units would be 38.5/1.5 which is approximately 26 jelly beans.
b. Assuming that the height of the jelly bean (when positioned laying on its longest side) is the same as the width (1 cm) then there would be 10 layers of jelly beans.
c. If there are 26 jelly beans on the bottom layer and the height is 1 cm, then the estimated number of jelly beans would be (26 jelly beans per layer)(10 layers) = 260 jelly beans. There is a similar answer when positioning the jelly bean with the smallest size on the bottom and using the longest side as the height (depending on when rounding occurs). The estimates are a little high considering that there is space between the jelly beans. They do not sit flush to one another.

Summarize the Mathematics

a) By dissecting the circle into smaller portions and rearranging the pieces, the circle became similar to a more familiar shape (parallelogram). The formula for area of a parallelogram was used to provide the structure for the formula and recognizing how those measurements related to the circle developed the formula for the area of a circle.
b) Both formulas use the area of the base and building layers (height) to calculate the volume.
c) The portion πr^2 is the area of the circular base. Multiplied by the height accounts for the multiple layers which provide the calculation for the volume.

Check Your Understanding

a) $(14\text{ in}^2)(12\text{ in}) = 168\text{ in}^3$
b) Student answers may vary. Two possible solutions are:

i. The hexagon can be divided into 6 triangles each with a base of 5 cm and a height of 4.3 cm. The area of a triangle = ½ bh and thus each triangle would be ½ (5cm)(4.3 cm) = 10.75 cm^2. The area of the hexagon would be 6(10.75 cm^2) = 64.5 cm^2.

ii. Dissecting the hexagon into smaller sections and rearranging them to form a parallelogram the area can be found by A=bh. The perimeter of the hexagon is 6(5 cm) = 30 cm. The base of the parallelogram would be ½ (30 cm) = 15 cm. The height of the parallelogram is 4.3 cm. The area of the hexagon is (15 cm)(4.3 cm) = 64.5 cm^2.

Investigation: Discovering Formulas for Area & Volume

Problem #1